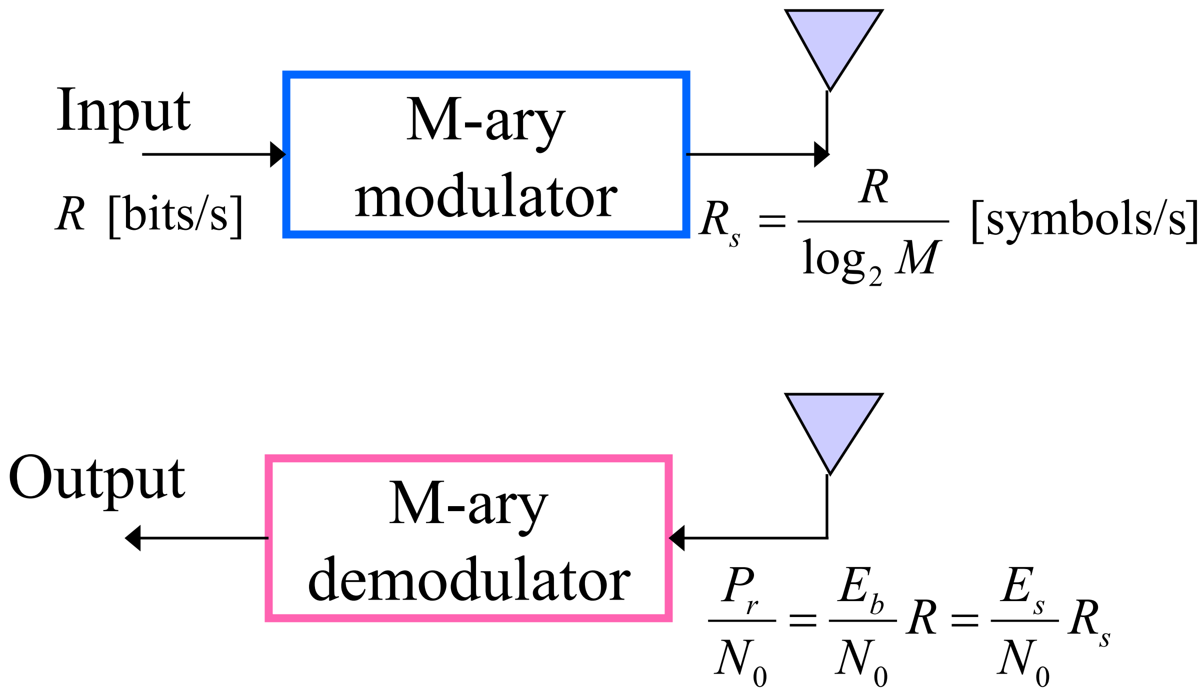
Design example of uncoded systems

Design goals:

- The bit error probability at the modulator output must meet the system error requirement.
- The transmission bandwidth must not exceed the available channel bandwidth.



Pr1:

$B = 4000$ Hz
$R = 9600$ bit/s
$P_b = 10^{-5}$
$P_r / N_0 = 53$ dB

Solution:

R/B = 9600/4000 = 2,4 > 1 => *bandwidth limited*

Pr/No [dB] = 10 log [(Eb.R)/No]

$$10^{5,3} = \left(\frac{E_b}{N_0}\right).R \Rightarrow \frac{E_b}{N_0} = \frac{10^{5,3}}{9600} = 20.783$$

10 log 20,783 = 13,17 dB = Eb/No [dB]

Na zaklade parametrov

$E_b/N_0 = 13{,}17$ dB a
R/B = 2,4 bit/s/Hz

najst bod v grafe: => **8 PSK**

Pr2: **Solution:**

$B = 45\ 000$ Hz
$R = 9600$ bit/s
$P_b = 10^{-5}$
$P_r / N_0 = 48$ dB

R/B = 9600/45000 = 0.213 < 1 => *power limited*

Pr/No [dB] = 10 log [(Eb.R)/No]

$$10^{4,8} = \left(\frac{E_b}{N_0}\right).R \Rightarrow \frac{E_b}{N_0} = \frac{10^{4,8}}{9600} = 6,57$$

10 log 6,57 = 8,17 dB = Eb/No [dB]

Na zaklade parametrov

$E_b/N_0 = 8,17$ dB a
R/B = 0,213 bit/s/Hz

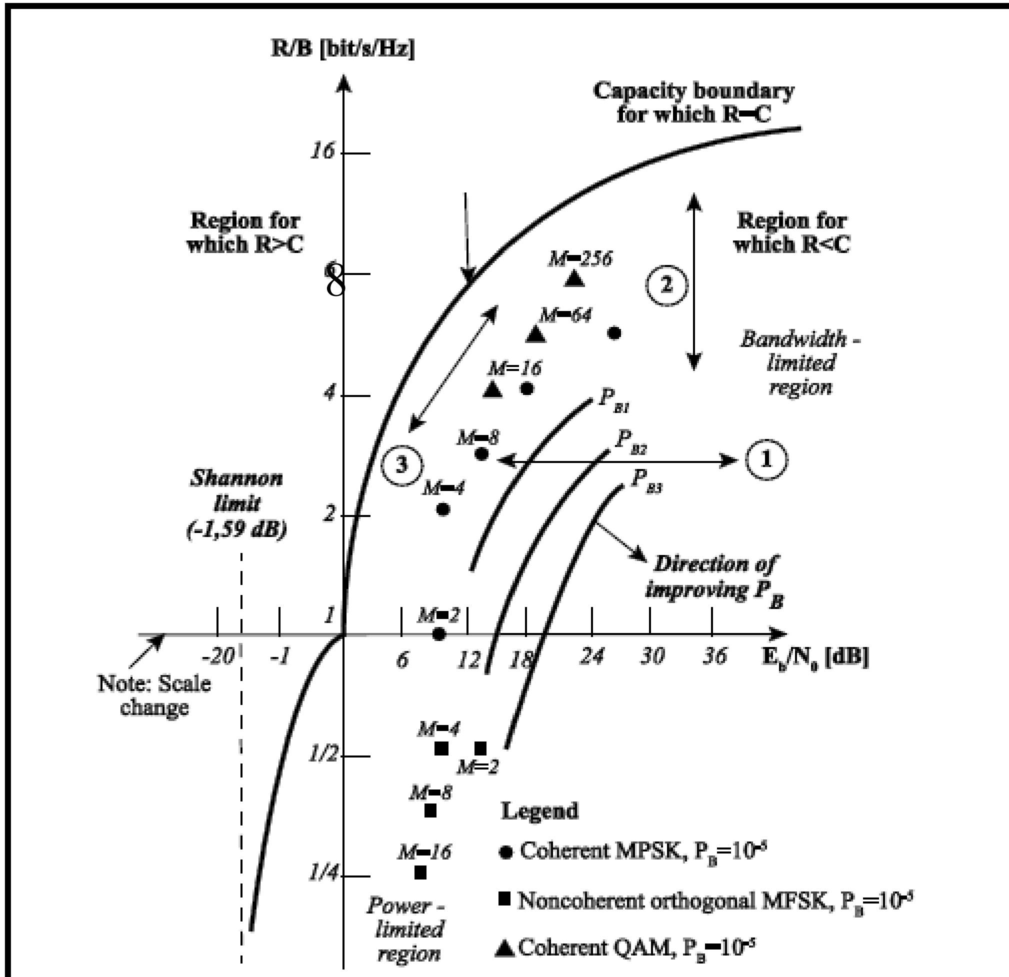
najst bod v grafe: => **16 FSK**

MPSK, QAM coherent

R/B = $log_2(M)$

MFSK noncoherent orthogonal

R/B = $log_2(M)/M$



Kontrola Pr1:

$$P_{BER-MPSK} \approx \frac{2}{\log_2 M} Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$

M=8 P=(2/3)Q(4,266)=6,63 E-6

Kontrola Pr2:

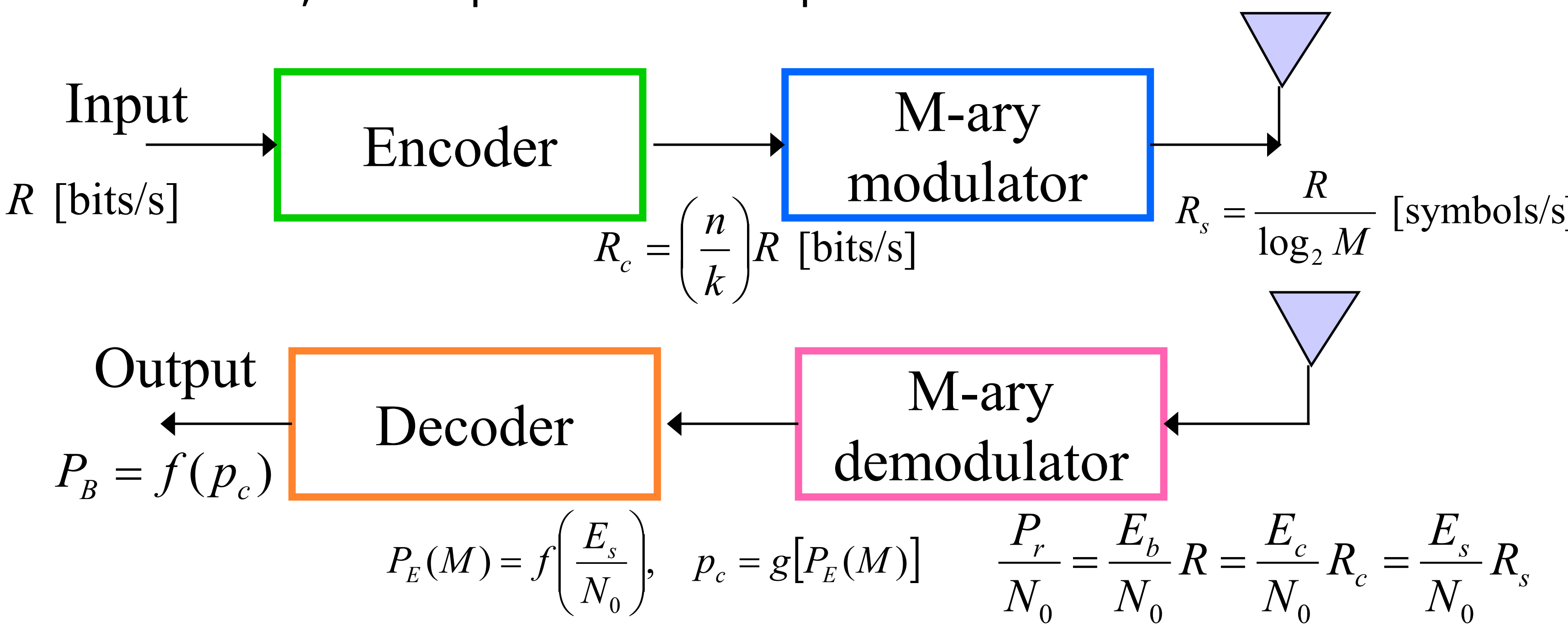
$$P_{BER-MFSK} \leq \left(\frac{M}{2}\right) Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

M=16 P=8Q(5,126)=11,84 E-7

Design example of coded systems

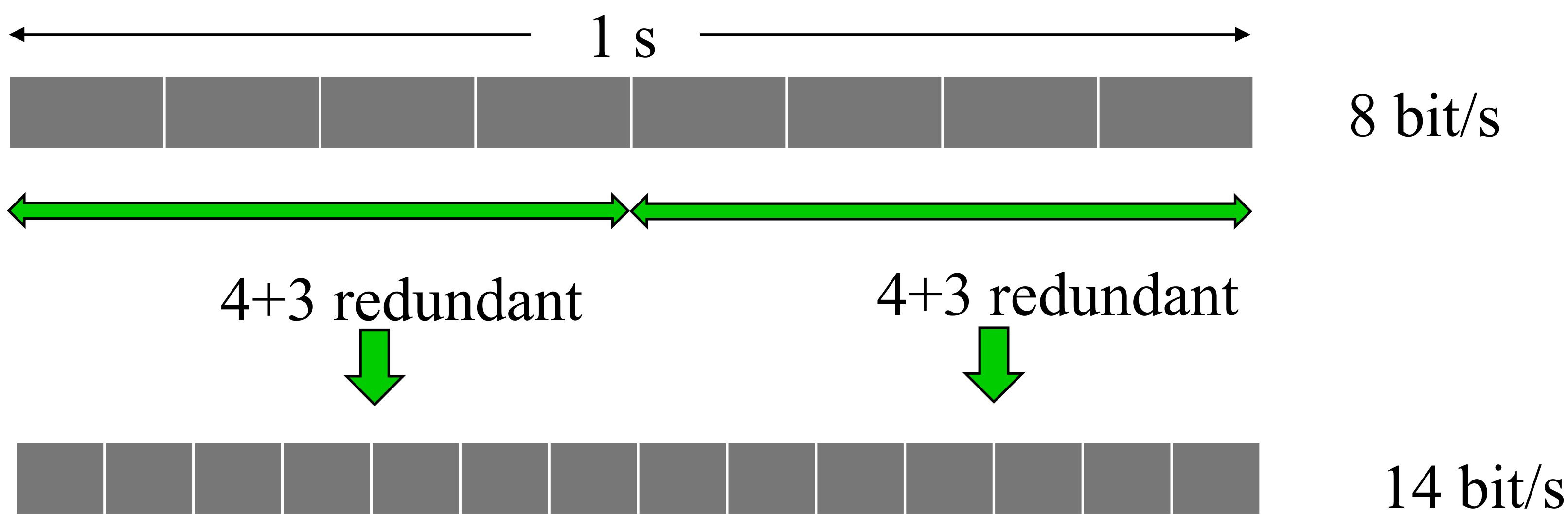
Design goals:

- The bit error probability at the decoder output must meet the system error requirement.
- The rate of the code must not expand the required transmission bandwidth beyond the available channel bandwidth.
- The code should be as simple as possible. Generally, the shorter the code, the simpler will be its implementation.



Objasnenie vztahu Rc=(n/k)R:

R=8 bit/s, kod (7,4), n=7, k=4, r=3



Rc = (n/k) R = (7/4) 8 = 14 bit/s

Design example of coded systems …

Choose a modulation/coding scheme that meets the following system requirements:

$$\text{An AWGN channel with } W_C = 4000 \ [\text{Hz}]$$

$$\frac{P_r}{N_0} = 53 \ [\text{dB.Hz}] \qquad R_b = 9600 \ [\text{bits/s}] \qquad P_B \leq 10^{-9}$$

$$R_b > W_C \Rightarrow \text{Band-limited channel} \Rightarrow \text{MPSK modulation}$$

$$M = 8 \Rightarrow R_s = R_b / \log_2 M = 9600/3 = 3200 < 4000$$

$$P_B \approx \frac{P_E(M)}{\log_2 M} = 7.3 \times 10^{-6} > 10^{-9} \Rightarrow \text{Not low enough : power-limited system}$$

The requirements are similar to the bandwidth-limited uncoded system, except the target bit error probability is much lower.

Design example of coded systems

- Using 8-PSK, satisfies the bandwidth constraint, but not the bit error probability constraint. Much higher power is required for uncoded 8-PSK.

$$P_B \leq 10^{-9} \Rightarrow \left(\frac{E_b}{N_0}\right)_{uncoded} \geq 16 \ \text{dB}$$

- The solution is to use channel coding (block codes or convolutional codes) to save the power at the expense of bandwidth while meeting the target bit error probability.

Design example of coded systems

- For simplicity, we use BCH codes.
- The required coding gain is:

$$G(\text{dB}) = \left(\frac{E_b}{N_0}\right)_{uncoded}(\text{dB}) - \left(\frac{E_c}{N_0}\right)_{coded}(\text{dB}) = 16 - 13.2 = 2.8 \text{ dB}$$

- The maximum allowable bandwidth expansion due to coding is:

$$R_s = \frac{R}{\log_2 M} = \left(\frac{n}{k}\right)\frac{R_b}{\log_2 M} \le W_C \Rightarrow \left(\frac{n}{k}\right)\frac{9600}{3} \le 4000 \Rightarrow \frac{n}{k} \le 1.25$$

- The current bandwidth of uncoded 8-PSK can be expanded still by 25% to remain below the channel bandwidth.

- Among the BCH codes, we choose the one which provides the required coding gain and bandwidth expansion with minimum amount of redundancy.

Design example of coded systems …

Bandwidth compatible BCH codes

Coding gain in dB with MPSK

n	k	t	$P_B = 10^{-5}$	$P_B = 10^{-9}$
31	26	1	1.8	2.0
63	57	1	1.8	2.2
63	51	2	2.6	3.2
127	120	1	1.7	2.2
127	113	2	2.6	3.4
127	106	3	3.1	4.0

Design example of coded systems …

Examine that combination of 8-PSK and (63,51) BCH codes meets the requirements:

$$R_s = \left(\frac{n}{k}\right)\frac{R_b}{\log_2 M} = \left(\frac{63}{51}\right)\frac{9600}{3} = 3953 \text{ [sym/s]} < W_C = 4000 \text{ [Hz]}$$

$$\frac{E_s}{N_0} = \frac{P_r}{N_0}\frac{1}{R_s} = 50.47 \Rightarrow P_E(M) \approx 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right] = 1.2\times 10^{-4}$$

$$p_c \approx \frac{P_E(M)}{\log_2 M} = \frac{1.2\times 10^{-4}}{3} = 4\times 10^{-5}$$

$$P_B \approx \frac{1}{n}\sum_{j=t+1}^{n} j\binom{n}{j} p_c^j (1-p_c)^{n-j} \approx 1.2\times 10^{-10} < 10^{-9}$$

Effects of error-correcting codes on error performance

Error-correcting codes at fixed SNR influence the error performance in two ways:

- Improving effect:
  - The larger the redundancy, the greater the error-correction capability
- Degrading effect:
  - Energy reduction per channel symbol or coded bits for real-time applications due to faster signaling.
- The degrading effect vanishes for non-real time applications when delay is tolerable, since the channel symbol energy is not reduced.